THE MATHEMATICAL ASSOCIATION OF VICTORIA

THE COMMON DENOMINATOR

4/24

TEACHING MATHEMATICS IN VICTORIA

INSIDE

David Howes, Department of Education (Victoria), and Jen Bowden, MAV CEO

In my 17 years working for MAV, including 18 months in the role of CEO, I have observed that members of our community share an interest in and commitment to high-quality mathematics teaching and learning. MAV members learn and work right across the education system. They are academics, administrators, consultants, school leaders, teachers, and preservice teachers.

Regardless of their career level or stage, MAV members distinguish themselves by their desire to be critically informed about how research can guide and enhance their practice. Many MAV members have experienced the benefits of participating in classroom research in Victorian schools where their own and their students' perspectives have helped to generate knowledge about what works.

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THE COMMON DENOMINATOR
The MAV's magazine published for its members.

Magazine 293, October 2024

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The Common Denominator is edited and produced by Louise Gray, Stitch Marketing.

Print Post Approved Publication No: PP100002988

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FROM THE PRESIDENT

Kerryn Sandford

Recently, I was interviewed by Channel Seven News regarding the 'crisis' of our falling NAPLAN results that was making the rounds of the various media cycles after the release of this year's data. In the interview, I was asked to provide a response to recent announcements of funding for a particular professional learning program to continue and then, quite quickly, asked to respond to questions about whether it was teachers who are to blame for the decline in student achievement in mathematics and numeracy.

I hope I was as clear in my response to these questions as I was in my own reaction to them.

At a time when teacher and school leader retention is at an all-time low and burnout and frustration levels high, it is not ok for teacher blaming narratives to be so prominent in the media.

Let's be clear, we know that there is variance in the capability and experience of the teacher workforce. We also know that we have a concerning number of teachers teaching 'out of field' in mathematics and numeracy largely due to the teacher retention and burn out issues mentioned above and the challenges that many schools are facing in getting qualified teachers to apply for positions in schools.

However, these issues are not due to any decline or lack of capacity, enthusiasm, commitment or passion of the educators that are turning up to work each and every day. Nor is it an issue with decline in the quality of teacher training. Blaming the teachers who are turning up for why students are not performing is only a recent phenomenon and one that is only likely to exacerbate the issue. Imagine blaming doctors and nurses for declines in life expectancy or survival rates for cancers!

The reality is that there are many factors that underly the trends that we see in the data around student performance and it is important that we recognise that responsibility for student outcomes is a shared one with teachers, families, policy makers and wider societal influence all playing a role.

In addition, I would like to encourage all teachers to be discerning about the reports about our NAPLAN data and what this data actually shows about our students' (and by association – at least as far as the wider media is concerned – our teachers') achievement and performance. When you look at Victoria's data, there is actually quite a lot to celebrate according to the Department of Education's release to schools on the 14 August 2024.

Given this, the reporting that we are seeing in the media is both unhelpful and inaccurate and the blaming of teachers for a crisis that may well not be occurring does nothing to address the issues facing our schools and our workforce.

Arguably, rather than blaming our teachers for the 'decline' in mathematics attainment, perhaps a stronger focus on the growing evidence of inequity and entrenched disadvantage (that is apparent in the recent release of our NAPLAN data) in our school system and the factors leading to this would be of greater benefit. Expecting teachers and schools to shoulder the full responsibility for student academic achievement neglects the long history of research and daily experience of the teacher workforce as to what hampers learning. It also neglects to consider the role of policy makers and the system engineers in considering this wicked problem.

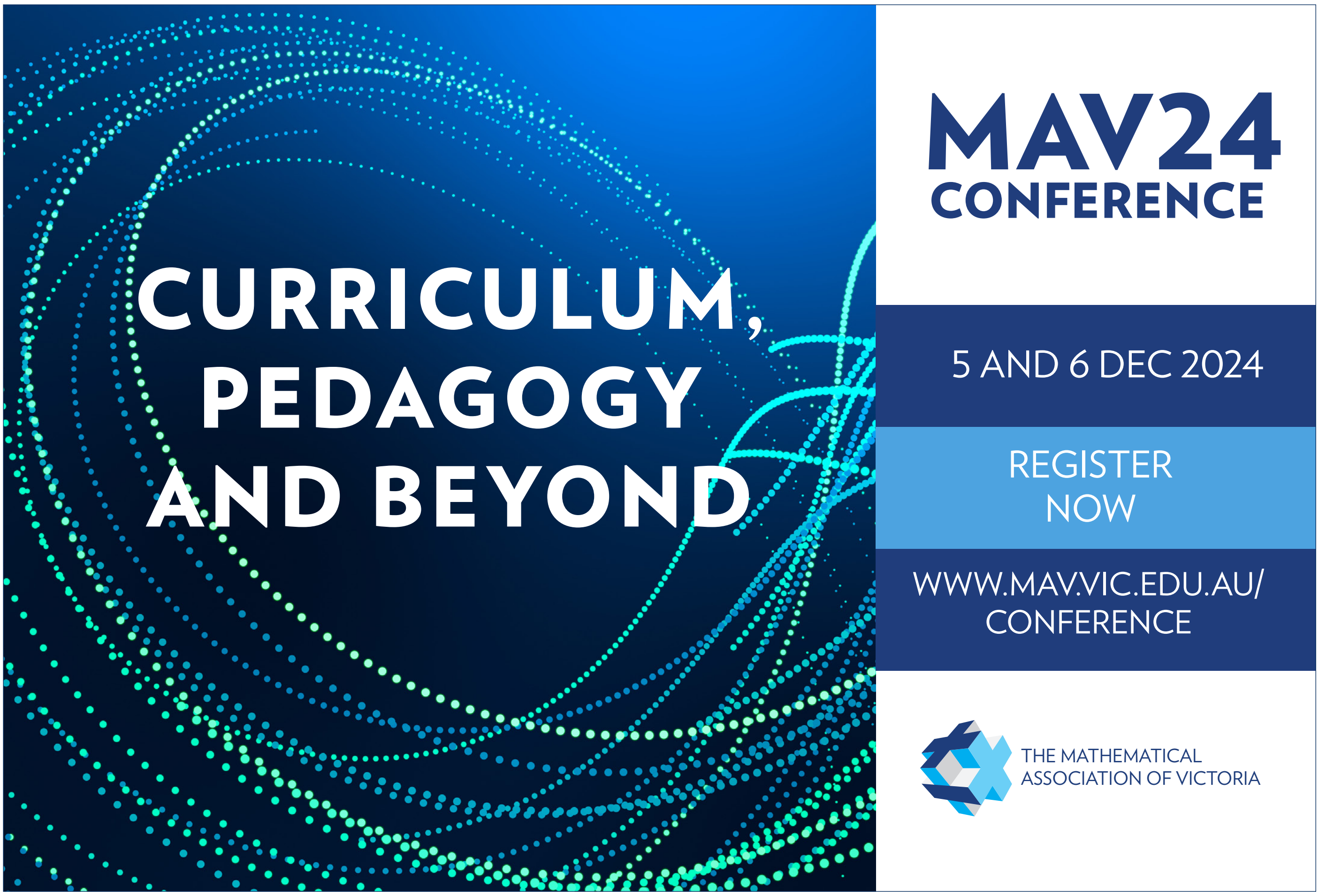
At MAV, we see and recognise the ongoing work of teachers to provide the best quality learning experiences for students across all sectors and all levels of schooling. We also work with teachers, leaders and schools across all sectors and levels of learning to help schools and teachers improve their practice to ensure that their students really are achieving their best. We will continue to engage in this work at the same time as advocating for teacher professional autonomy, learning and growth in ways that support all teachers of mathematics to thrive and improve and to help encourage innovation and progression towards a more equitable and fair education system.

I hope to see you at MAV24, our annual conference which will be held on 5 and 6 December at La Trobe University. It's our best chance to come together, network and share our collective learnings.

UPCOMING MAV EVENTS

For more information and to reserve your place at any of the events below, visit www.mav.vic.edu.au.

EVENT	DATE	YEARS	PRESENTERS
Algorithms that make thinking, risk-taking and narratives visible (part 1)	14/10/24 (Virtual)	F–10	Colin Chapman
Algorithms that make thinking, risk-taking and narratives visible (part 2)	21/10/24 (Virtual)	F–10	Colin Chapman
VTLM 2.0 and VCM:2.0	24/10/2024 (Virtual)	F–10	Peter Burrows
VTLM 2.0 and planning	29/10/2024 (Virtual)	F–10	Aylie Davidson
Springboard series: Unlock AI's transformative power	30/10/24 (Virtual)	5–VCE	Chris Bush
Charting the course: Mapping in early childhood education	30/10/24 (Virtual)	Early years	Dr Rachel Pollitt
Banging down the doors to get in: How to get your students excited about learning mathematics	12/11/2024 (Virtual)	5–10	Thomas Moore
VTLM 2.0 and pedagogical approaches	27/11/2024 (Virtual)	F–10	Peter Sullivan
MAV annual conference: Curriculum, pedagogy and beyond	5/12/24 6/12/24	All	Various



TEACHING MATHEMATICS IN VICTORIA

David Howes, Department of Education (Victoria), and Jen Bowden, MAV CEO

CONT. FROM PAGE 1.

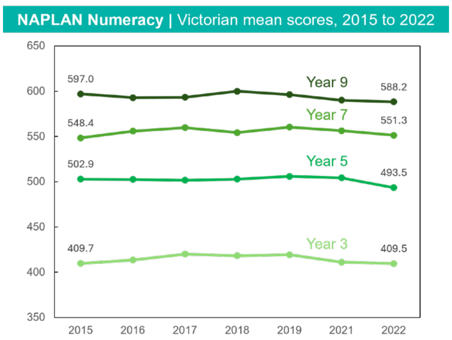
Sadly, polarising instructional debates, seem to be undermining teachers' professional expertise and right to choose from a variety of pedagogies that can meet their students' learning needs. Together with the MAV Board, I have been reading widely to self-educate about the origins of these debates and the reasons why reputable mathematics education researchers are concerned about naming any one instructional practice as 'best'.

As part of my commitment to gather and reflect critically on a range of perspectives, I recently met with Deputy Secretary Dr David Howes to discuss our state's mathematics achievement data, the need to support teachers working in and out-of-field in mathematics classrooms, and the strategies he believes can help to deliver more equitable and excellent outcomes for all students. David's insights follow here:

The level of mathematics attainment across all school sectors in Australia, including Victoria, should be a matter of collective focus.

The 2024 NAPLAN results were pleasing to the extent that in Victoria we did see the mean numeracy scores go up (albeit very marginally!) from the 2023 results for Years 5 and 7. But this was offset by a slight decline in the results for Years 3 and 9. See Figure 1.

The improvement in Years 5 and 7 was pleasing, even if it was marginal, because it reversed the trend of either stagnation or steady decline in our numeracy performance.



But one of the most concerning aspects of our current attainment levels in mathematics remains, which is the performance gap between advantaged and disadvantaged students. See Table 1.

Numeracy	All students		Disadvantaged students*	
	2023	2024	2023	2024
Year 3	417.7	413.4	362.3	359.3
Year 5	494.3	497.3	435.4	438.7
Year 7	544.0	545.5	481.1	483.5
Year 9	574.0	571.1	519.7	512.2

Table 1. 2024 NAPLAN results (as defined by lower parental education attainment.)*

These challenges are not unique to Victoria – they are evident across all jurisdictions in Australia. But they are challenges that we in Victoria need to address.

The Minister for Education and Deputy Premier, the Hon Ben Carroll, recently announced revisions to the Victorian Teaching and Learning Model (VTLM). These will have implications for the way mathematics is taught in Victorian government schools.

While the revisions reflect the practices in teaching mathematics already in place in many government schools, for others alignment with the revised model will require a shift in current practice. The revised VTLM is shown on page 5, and can be downloaded online.

The foundation of the revised VTLM is a model of *learning* because an effective model of teaching depends on a common, evidence-based model of learning.

The revised VTLM, which has drawn substantially on recent similar work by the Australian Education Research Organisation (AERO), sets out four key elements that constitute and enable the learning process:

- Attention, focus and regulation
- Knowledge and memory
- Retention and recall
- Mastery and application.

It then sets out four elements of effective teaching.

1. Planning calls out the importance of a whole-school approach to teaching and learning. This means a collaborative approach is taken to developing a common teaching and learning program, and this program is then followed consistently by all teachers in a school.

Underpinning this is a basic proposition that the collective expertise of teachers in designing a common teaching and learning approach is likely to be more effective for more students than multiple programs designed by individual teachers working alone.

This does not diminish the professionalism or expertise or agency of individual teachers. Because whatever program is developed, there will always be students who either learn at a quicker or slower pace than most of their peers.

This is precisely where the expertise and agency of individual teachers is most effectively deployed, in making the countless micro-decisions each day about what intervention will best support this particular student in this specific context at this exact moment in time.

2. Enabling learning foregrounds the necessary – albeit not sufficient – condition of effective teaching which is an enabling classroom environment for learning that develops students' capacity for self-regulation and supports the development of student efficacy.

3. Explicit teaching is the key point of connection between learning and teaching. Explicit teaching involves collaborative planning to ensure the teaching and learning is both effectively sequenced – including, for example, concrete to pictorial to abstract progressions – and paced to reduce the risk of cognitive overload and consequent confusion and learning frustration.

Explicit teaching does not mean there is no place for inquiry-based learning and problem solving.

It certainly does not mean there is no place for supporting and enabling student curiosity and reasoning and questioning. But it does mean that privileging the importance of planning to ensure students first develop an understanding of the concepts and procedures they will need to use to investigate and propose solutions to a range of problems before they are presented with the problem.

4. Supported application, that critical opportunity for students to apply their new knowledge to a wide range of increasingly complex problems.

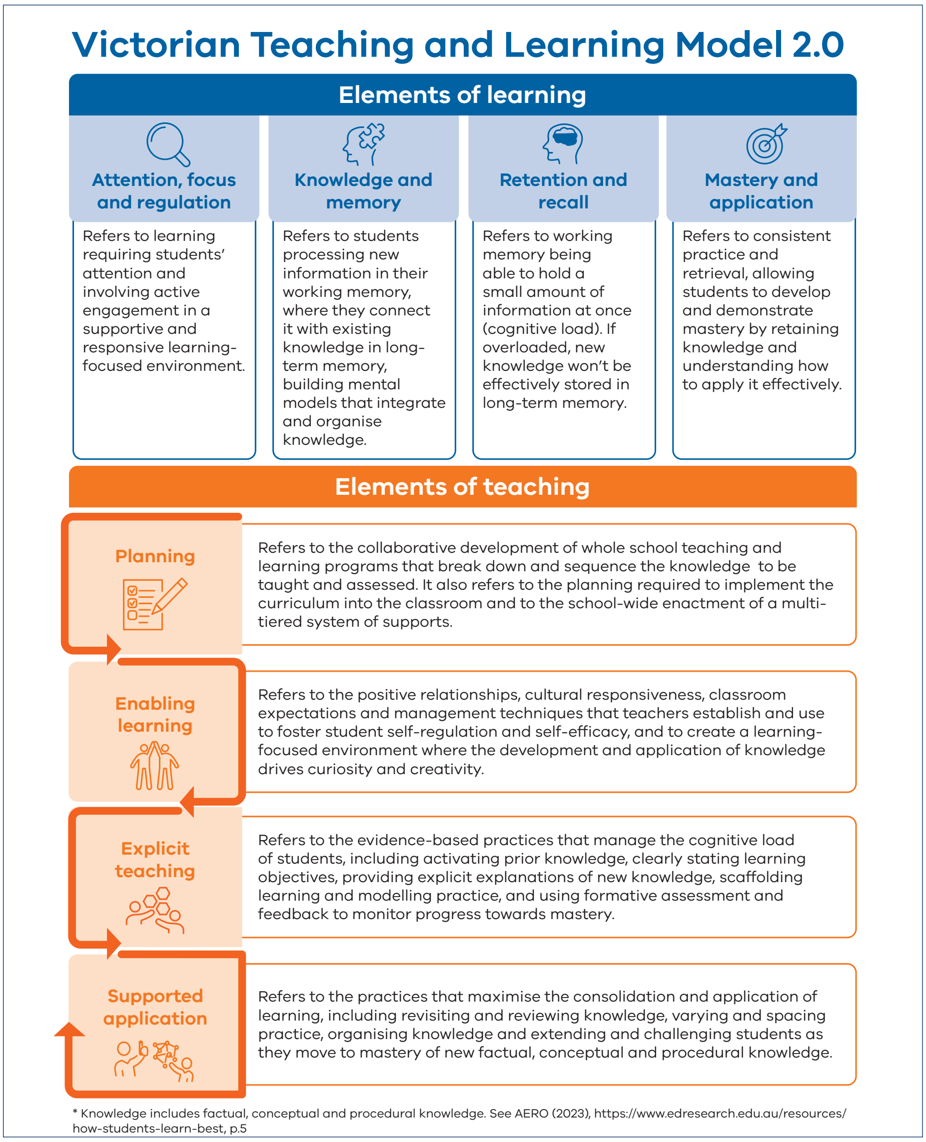
The revised VTLM is not an end in itself. Effective teaching is not an end in itself. Both are means to an end that sees as many students as possible developing the proficiency in mathematics that will both provide them with the pragmatic capacity to manage the numeracy requirements for participation as a full citizen in our contemporary and future communities, and provide access to the joy and wonder of mathematics.

It took a Year 3-4 class in an inner-city school to remind me of the conceptual purity of mathematics. Recently, I worked for a day as a relief teacher at one of our most disadvantaged primary schools. The daily outline left included a video link that explained two and three dimensions. The class and I watched in increasing bafflement.

I couldn't follow the explanation and neither could any of the students. So I switched it off, brought all the students to the front of the room, sat down on one of the small chairs, and had a go at explaining two dimensions and three dimensions.

In a moment, I was taken back many years earlier to when I had been a student in a Year 7 maths lesson and we had to memorise some definitions. While it was generally a dreary task, I vividly recalled turning over and over in my mind the abstract perfection of the sentence: *A point marks a position but has no magnitude.*

It was a reminder that mathematics is beautiful in its conceptual purity and that giving all students access to that beauty is one of the great responsibilities – and delights – of school education.



The introduction of the revised VTLM alone is not going to immediately and miraculously lift numeracy attainment levels nor engender a sudden love of numbers in every student.

There are other deep and complex issues to address including a widespread culture of acceptance of being 'no good at maths' throughout our community, associated 'maths anxiety' that is such a barrier to learning, and long-standing teacher shortages in the areas of mathematics and science. But it does give us an excellent framework to ensure that the outstanding teaching practices in many of our schools become the teaching practices used in all our schools.

As part of MAV's vision to develop confident, capable and engaged lifelong learners of mathematics, we aim to support all Victorian teachers to be what your students need you to be: the best possible teacher of mathematics. We will continue to offer a range of high-quality resources and professional learning opportunities, including on-demand professional development modules, regional conferences, online community discussions, and targeted in-school consulting. If your school is interested in targeted professional learning or consulting to support curriculum planning with one of our mathematics education experts to effectively implement the VTLM 2.0, please contact consulting@mav.vic.edu.au.

THE WORLD OF THE CHILD

Doug Clarke, Emeritus Professor of Mathematics Education, Australian Catholic University

DRAWING INSPIRATION FROM THE WORLD OF THE CHILD: MAKING MATHEMATICS RELEVANT, WORTHWHILE AND ENJOYABLE

It is widely acknowledged that as students progress through schooling, many come to dislike mathematics, seeing it as irrelevant (Grootenboer & Marsham, 2016). As part of the Australian Mathematics Curriculum and Teaching Program (Lovitt & Clarke, 1988), teachers in all Australian states and territories were asked to identify their concerns about the teaching of mathematics in the middle years.

Common responses were: mathematics was seen by many students as boring and irrelevant; little thinking was involved; the subject was too abstract; student exhibited a fear of failure; too much content was covered in too little depth; assessment was narrow; and it was a huge challenge to meet the needs of a wide range of abilities.

Readers can judge whether the same comments can be made of mathematics as it is experienced today, particularly in relation to the worrying recent trend towards an over-reliance on 'explicit teaching' and 'direct instruction.'

But the topic of this article relates to responses to the first criticism (by students and teachers) that school mathematics is seen as boring and irrelevant.

There have been many calls over the years for teachers to make mathematics more relevant for students. These calls have been expressed in many ways. Possibly the greatest benefit of relevant or contextualised tasks is that students see the ways in which mathematics can help us make sense of the world (see, e.g., Meyer, et al., 2001).

Tout (2014) described the way in which mathematics is typically presented to students in secondary classrooms (teacher explanation, practice, and possibly some application). He contrasted this with other models, such as the international assessment framework in the Program for International Assessment (PISA).

In the PISA situation, Tout argued, the starting point is the problem in its context, not the mathematics, and the first step is formulating situations mathematically by identifying how to apply and use mathematics to the problem being posed in the real world. He proposed that we need to teach students to identify and extract the mathematics from the messy, real-life situations that they are likely to face.

Meyer et al. (2001) argued that high quality contexts:

- Support the mathematics, not overwhelm it.
- Should be real, or at least, imaginable to the student.
- Should be varied, not repeated over and over.
- Should result in real problems to solve.
- Should be sensitive to cultural, gender, and racial norms and not exclude groups of students.

Care must be taken in choosing tasks which have elements of relevance but may not be of interest to students. Boaler (1993) noted that tasks which are extracted from the real world of an adult (e.g., household bills) 'may take on the status of another mathematical exercise' (p. 14), and do not really allow students to appreciate the reality of the task.

The argument which underpins this article is that possibly the best place to start in making mathematics relevant for young children is to start with the things in which they are interested.

DRAWING INSPIRATION FROM THE WORLD OF THE CHILD

Take a moment to think about the children you teach. What are the things which currently fascinate them? Having had the chance to ask primary teachers in recent years to nominate the things which fascinate the children they teach, the following list provides a sense of the things which emerge:

- Minecraft
- Bey Blades
- Unicorns
- Beany Boos
- Marvel characters
- Animals: bugs, penguins, lizards
- Anything to do with their teacher's own children
- Slime
- Squishies
- Jumping off bridges into rivers
- Sandpits
- Fortnite
- Baby names
- Fishing
- JoJoBows
- Tech deck
- Loom bands
- Astronomy
- Olympics
- Sport: basketball, netball, BMX etc.
- Lego
- Flossing – the dance phase

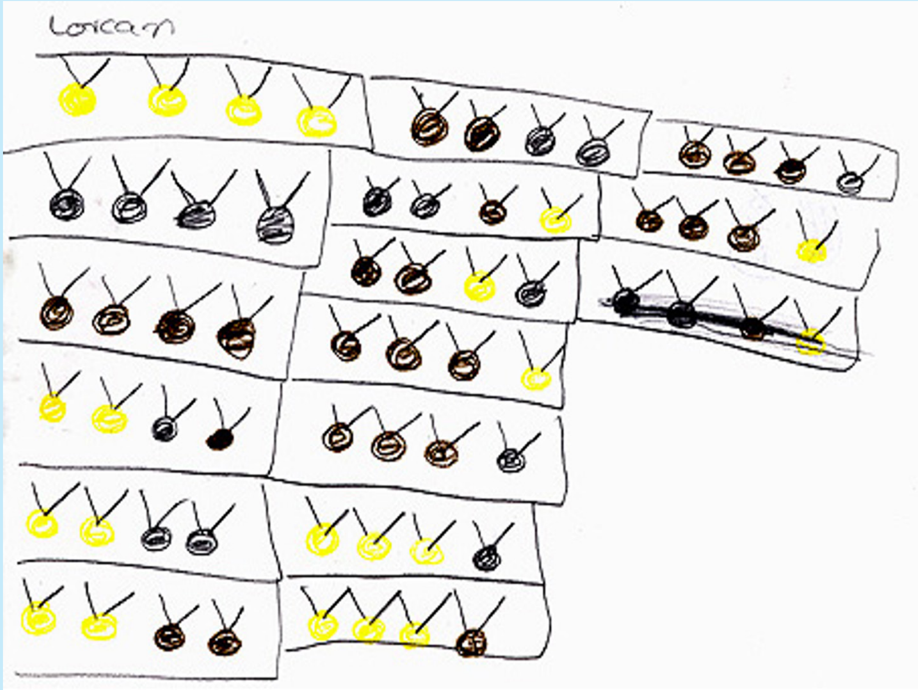


Figure 1. A student's solution to the Olympic medal problem.

- AFL trading cards, Pokemon cards
- Ooshies
- Jumping over creeks
- Motorbikes
- Taylor Swift

To give an example of building on a student interest/fascination, consider the last item in the list, Taylor Swift. Teachers will know 'all too well' how her 2024 February tour in Australia captured the imagination of so many people. Michael Minas, in an article in *Prime Number* (see Minas & Clarke, 2024), offered a whole range of examples of how teachers could plan mathematical experiences which built on this enthusiasm, *at the time when interest was greatest*.

In the remainder of this article, I will use four things which fascinated students I have worked with (or my six grandchildren) and discuss how these interests can form the basis of highly worthwhile classroom activity.

FOUR EXAMPLES OF BUILDING ON STUDENTS' FASCINATIONS

The Olympics

The Paris Olympics provide so many opportunities to build on students' interests. Students can build measurement concepts and skills to get a sense of the extraordinary feats of athletes. For example,

- Students can measure out 8.95 metres (the men's world long jump record), or 18.29 metres (the men's world triple jump record - about the length of three typical classrooms!), or 2.09 metres (the world women's high jump record)
- During the Olympics, students can be introduced to the idea of a table of medals. Using a real medal table, but not showing it to students initially, they can be told that New Zealand, say, at that stage of the Olympics has four medals. They can then be challenged to find out how many different ways a country might have achieved four medals. Figure 1 shows one student's solution.

Baby names

There will be students in the class whose parents are expecting (or have just had) a baby. There is always fascination around the choice of baby names. Students could determine the ten most common first names for boys and girls in their school. They could then consider the most common baby names for the previous year in Victoria, Australia.

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THE WORLD OF THE CHILD (CONT.)

Doug Clarke, Emeritus Professor of Mathematics Education, Australian Catholic University

For 2023, these were:

	Girls	Boys
1	Amelia	Oliver
2	Charlotte	Noah
3	Olivia	Henry
4	Mia	Leo
5	Isla	Charlie
6	Hazel	Luca
7	Matilda	Jack
8	Ava	Archie
9	Grace	Hudson
10	Ella	Thomas

How does this list compare to the one they generated? Why might that be?

How big a creek can you jump?

At the end of our street is the Ferny Creek. The width of this creek varies considerably over time. My grandchildren love the challenge of jumping the creek.

My experience is that many students are interested in jumping creeks, leading to a discussion about the size of the widest creek they can jump. This can lead into an investigation where students work in pairs, with chalk and one length of streamer each.

Their challenge is to go outside and return to class with a streamer which exactly matches the largest creek they can jump. This can lead to all sorts of comparisons of lengths, uses of formal units, using one metre rule to measure a length greater than one metre, and so on. (A version of this lesson is detailed in Clarke & Roche, 2014.)

Sandpits

Most children love sandpits. I was digging holes in a huge sandpit (actually a long jump pit) with my eight year-old grandson. We started to talk about what would happen if we kept digging until we got through the centre of the Earth and came out the other side. Where would we come out? This led to several weeks of exploration of globes and maps to answer this question. Along the way, there was a growing understanding of Northern and Southern Hemispheres, the proportion of sea and land on the face of the Earth, latitudes and longitudes, and even great circle routes (a topic I first learned about in Year 12).

These are just a few examples of taking students' fascinations/interests and building on this interest to create worthwhile mathematics experience. Teachers can work together to plan such experiences, but also encourage the students to pose problems which they personally wish to solve, in relation to a particular topic of interest.

A FINAL NOTE

I often find myself working in a school, and using equipment which, while not expensive, takes some time to put together.

After using the equipment with one group of students, I often say to teachers, 'I'm around all day. If you would like to use this with your class, then please go ahead.' I often get the response, 'Oh, that's okay, we're not doing geometry [or probability or fractions or …] at the moment.' Personally, I see this as a lost opportunity. Perhaps we need to be flexible in our approach to the mathematics curriculum.

In the same spirit, out of the blue, sometimes a child will come to class with a question or share a personal story that reveals a fascination which mathematics has the potential to bring to life. The teacher then must make a call - do I go with this or stick with what I'd planned for the maths lesson for the day? My experience is that responding to interests of students is likely to be quite an efficient way to address a number of mathematics content areas in the one investigation. I believe that it is almost always worth 'going with the flow'.

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Doug Clarke is a keynote speaker at MAV's annual conference, the leading mathematics education event in Victoria. Registrations are essential, book your place now at www.mav.vic.edu.au/conference.

PLAY-BASED MATHEMATICS

Dr Rachel Pollitt, Research Fellow, The University of Melbourne

PLAY-BASED MATHEMATICS: LEARNING AND ASSESSMENT

Young children often explore complex mathematical ideas across many curriculum areas in unique and varied ways (Pollitt, et al.2015).

Three- to four-year-old children also demonstrate understandings of math which can cross domains, be interrelated, and encompass multiple areas of mathematical thinking, sometimes, all at once. These understandings are typically enacted by children across every aspect of their learning environments, including everyday routines, transitions, play, rituals and being on Country. At times, observations of children's mathematics explorations can be difficult to untangle. In short, children do not necessarily explore mathematics ideas separately or in isolation (Pollitt, Cohrssen & Seah, 2017; 2018; 2020).

Challenges can arise when we are unsure of what to look and listen for as evidence of children's demonstrated mathematics knowledge. To usefully enact a planning cycle and promote children's mathematical learning requires skill. We need to have a degree of mathematics knowledge to understand what children already know and are ready to learn next.

This requires intentionality, and place-based teaching strategies which create culturally and socially safe spaces for children, families, communities and kinship systems.

It is important to consider what maths is in each ECE context and what is being assessed as evidence of children's strengths in mathematics explorations and learning. An example of this comes from an observation, documented by a student undertaking a Master of Early Childhood Education (UoM). Seke kindly shared his observation with me during discussions about assessment:

> Tia (4.5 years) is carefully constructing a line from a tree and continues making this along the ground. She is using sticks, long pieces of bark and shorter sticks to bend the line around anything in the way of her work. She is focused on her goal.
>
> Seke: 'Tia would you like to tell me about this?', and Tia says, 'No, not yet…' She continues working, carefully making sure the end of each stick is touching. The sticks have run out and Tia stands at the end of the line looking around. She begins to collect rocks, dropping them into a large tub. The rocks make a sound. Two children come and join her. Tia says, 'That one is too big, it needs to be like this…' showing them an example. One child says, 'OK' and heads off toward a rocky area beside the small creek.
>
> The children are focused. Tia calls out, 'Put them in here', and points to the tub on the ground. Children keep adding rocks until it is full and when Tia tries to lift it up, Seke can see it is heavy. A child comes over with another tub and starts to transfer some of the rocks from one tub to the other. 'Wait' says Tia, 'We need small ones first'. She tips the tub over, and the children start sorting the rocks by size.
>
> There is a lot of discussion about shape, weight, size, even 'that one is dirt', as the children begin comparing rocks and washing them in puddles. Tia starts sorting again, by colour and size. This takes time. In the end, there are three piles of rocks, 'smallest', 'middle', 'big ones', plus 'big white ones' and 'small white ones.' See Figure 1.

Figure 1. Tia shows the rocks to Seke for sorting.

The children start to add the smallest rocks to the end of Tia's line, then the 'middle' and 'big ones'. A child calls out 'How much more?' Tia demonstrates by holding up her hands like a cup saying, 'This much'. They keep working until there is a complete line from the tree to the edge of a tree stump near the quiet yarning space. Tia looks at the line and says, 'Done'.

Figure 2. A young child washes tiny pebbles – some dissolved because they were dirt.

The evidence is compelling, as children's curiosities lead them to explore size, number, shape, weight, measurement (volume, length), comparison, matching, categorisation by attribute, mapping, (the line began to map the space into 'places' where *Marngrook* was played) direction collaboration, working memory prediction and estimation to explore problem-solving strategies. What did you notice as evidence of mathematics learning in this observation? How would you assess this?

I wondered about the rules of where to play *Marngrook* and how this collaborative problem-solving effort eventuated, sustaining the interests and curiosities of 3 – 5year old children for over an hour.

Tia knew 'why' the line was made, 'To stop the ball hitting here (where children were playing) and in the plants'. This was part of an ongoing group discussion about how to share the environment safely. Tia was going to communicate this new 'rule' with everyone. She discussed this with Seke as they walked beside the line together. Using words like, 'fair', 'safe', 'respect', 'one side', 'divided', 'bendy bark', 'longer sticks', 'heavy', 'lots', needing 'more rocks' and 'caring for plants and us', Tia explained her detailed process. Seke asked her why each rock, stick and piece of bark had to be touching and Tia said, 'To make the line'. He asked, 'What happens if they are not touching', and she said, 'It is not a line'. There was a pause. Tia said, 'Look' and lifted up a rock. 'Now it is two lines, not one', she put the rock back and ran off.

Thinking about an Early Years Framework and each aspect of a Planning Cycle (Figure 3.), what do you identify as evidence for ongoing exploration? How does this inform planning? In this example, mathematics has a broader application than being an isolated lesson in a classroom or one-off play episode. Mathematics can promote children's connection to their environments, strengthen their care for each other and the land they are on.

Children's thinking and imagination are enhanced and not held within the socio-cultural environment of physical space, resources and meaning-making processes which are outside of their determination (Prinsj et al., 2022). Through research and practice evidence we know that children and educators' wellbeing, health and cognitive functioning are enhanced when time is spent engaging with nature (Dankiw et al., 2020) and connecting to the land.

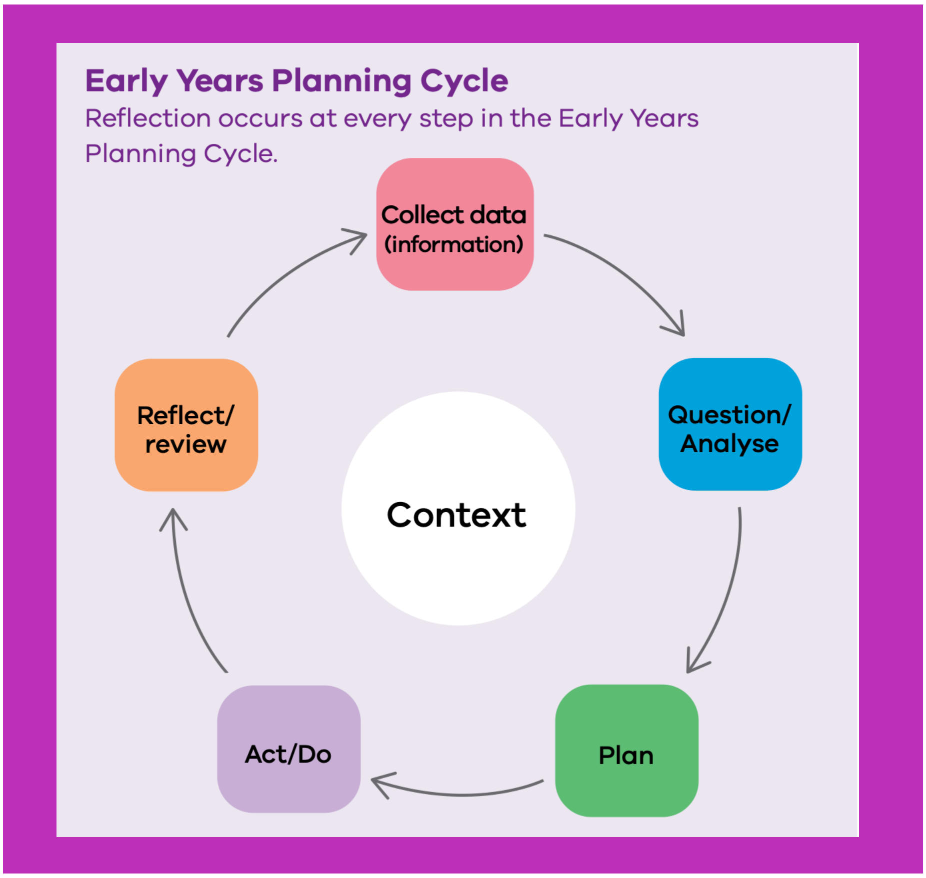

Figure 3. The Victorian Early Years Learning and Development Framework (DET, 2016)

Young children show us that how we historically think about teaching mathematics across the early years (0-8) is up for review. Children teach us that maths concepts can overlap, are at times tangled, confusing and confused. Mathematics learning is a process, which can be reflected on with children, and untangled to capture their ongoing interests. Seke focused on 'being' and engaging in Tia's process as it unfolded, promoting her wellbeing and confidence as she experimented with maths concepts, language and problem-solving strategies.

HOW DO YOU PROMOTE CHILDREN'S MATHEMATICS LEARNING? WHY?

Unpacking children's mathematics knowledge and promoting their thinking are an essential aspect of every child's right to participate in early years mathematics learning which contingently responds to their socio-cultural context, family, kinship systems, narratives of wellbeing and development.

REFERENCES

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Prins, J., van der Wilt, F., van Santen, S., van der Veen, C., & Hovinga, D. (2022). The importance of play in natural environments for children's language development: an explorative study in early childhood education. *International Journal of Early Years Education*, *31*(2), 450–466.

For further references and readings see Dr Pollitt: orcid.org/0000-0002-7831-2026.

FEYNMAN: PHYSICIST, SAFECRACKER

Roger Walter

Feynman's Los Alamos ID badge, copyright Los Alamos National Laboratory. Used with permission. https://commons.wikimedia.org.

Richard Phillips Feynman was an American theoretical physicist. Among other things, he is known for his work in quantum mechanics, liquid helium and particle physics, for which he developed Feynman diagrams. He jointly won the Nobel Prize in 1965. He also worked on the Manhattan project, developing the atomic bomb, before he had even completed his graduate degree.

During this time, which he found rather boring, he amused himself by cracking the codes of the combination locks on cabinets and desks. He began by picking the ordinary tumbler locks using a paper clip and screwdriver, which he found easy, after befriending a locksmith he knew at work. He would often leave a note as a prank, unsettling the unfortunate victim, thinking a spy must have gained access. Besides the challenge, part of his motivation was to alert his superiors to security flaws in the locks.

Feynman wrote the following in his essay *Safecracker Meets Safecracker*:

> To demonstrate that the locks meant nothing, whenever I wanted somebody's report and they weren't around, I'd just go in their office, open the filing cabinet, and take it out. When I was finished I would give it back to the guy: 'Thanks for your report.'
>
> 'Where'd you get it?'
>
> 'Out of your filing cabinet.'
>
> 'But I *locked* it!'
>
> 'I *know* you locked it. The locks are no good.'

As a result, the officials installed better locks (see page 13), but Feynman was still able to crack them with little effort. He claimed:

> I opened the safes which contained all the secrets to the atomic bomb: the schedules for the production of the plutonium, the purification procedures, how much material is needed, how the bomb works, how the neutrons are generated, what the design is, the dimensions – the entire information that was known at Los Alamos: *the whole schmeer!*

STUDENT INVOLVEMENT

You will probably find that many of your students will be interested in how to crack codes for combination locks. You could use this as a basis for an investigation for your class, perhaps during one of those 'passionless moments', for example, towards the end of a Friday afternoon. I have therefore included some possible questions you could ask your students, depending on their levels of understanding.

The new locks involved inputting three numbers between 0 and 99, using a rotating dial. A possible passcode could be 97 34 04 or 02 63 17 etc.

The obvious first question is, how many different combinations ae possible? Students familiar with permutations and combinations will quickly calculate $100 \times 100 \times 100 = 1\,000\,000$ different possibilities, or maybe $10^6 = 1\,000\,000$. More junior students might work out that if every combination of numbers was written as a single number, you will have a million numbers, going from 00 00 00 to 99 99 99.

THE TIME FACTOR

The logical next question is, how long would this take? If it took a second to try each combination, the calculation is:

$1\,000\,000 \div 60 \div 60 \div 24$

This works out to be 11 days 13 hours 46 minutes 40 seconds. Far too long! No doubt this length of time was considered sufficient to make the new locks secure.

This may be a good place to discuss a suitable accuracy for this answer. It would probably be sufficient to say 'about a week and a half if you don't sleep, or over two weeks if you do, allowing a third of a day for sleeping'.

This figure assumes you get the correct combination with your last trial. Intuitively, all students should be able to see that the *average* time would be half of this, or a little under six days (a week allowing for sleep), sometimes more, sometimes less but still way too long. This could be a good place to discuss with students just how big a million is!

The administrators reasoned that the new locks could not be opened easily, but Feynman was to prove them wrong.

How did he do this (so quickly)? After tinkering with the locks, Feynman noticed that the safes weren't mechanically perfect and had a tolerance of ± 2 on each number. For example if 17-42-49 was the set passcode, 15-44-47 and 19-40-51 would also open the lock. He only needed to check every fifth number, or 20 different numbers on each set of two digits. How many possibilities are there now? Using permutations, $20 \times 20 \times 20 = 8000$.

There is also a simple solution for junior students. The first number now has 20 possibilities rather than 100, one fifth as many, so just considering the first (double-digit) number, the total decreases to $1\,000\,000 \div 5 = 200\,000$. The same applies to the second and third numbers, giving $200\,000 \div 5 \div 5 = 8000$.

How long? 8000 seconds is about 2 hours, so on average, 1 hour. Not too bad.

Feynman also noticed that most people will set their safe to some kind of a date in the past. With 30 days in a month (sometimes 31), 12 months and, say, a year between1954 and 2024, giving allowing for the ±2 tolerance, there are 3 months, 14 years and 6 or 7 days to check (allowing for some 31 day months). Americans put the month before the day, but this does not alter the result.

This gives about $6.7 \times 3 \times 14 = 280$ seconds. At 1 second for each try, the maximum time is about four and a half minutes, with an average of just over two minutes – quite achievable! One author has claimed you could 'easily' test 1000 combinations in half an hour, working out at about eight and a half minutes, average time just over four minutes.

Students may suggest another (not quite so accurate) approximation of 365 (for day and month), and 14 years, giving $365 \div 5 \div 5 = 15 \times 14 = 210$ seconds. Why is this lower than the previous calculation?

I suggest as far as practical you get your students to do these calculations themselves. This would make the exercise more meaningful for students.

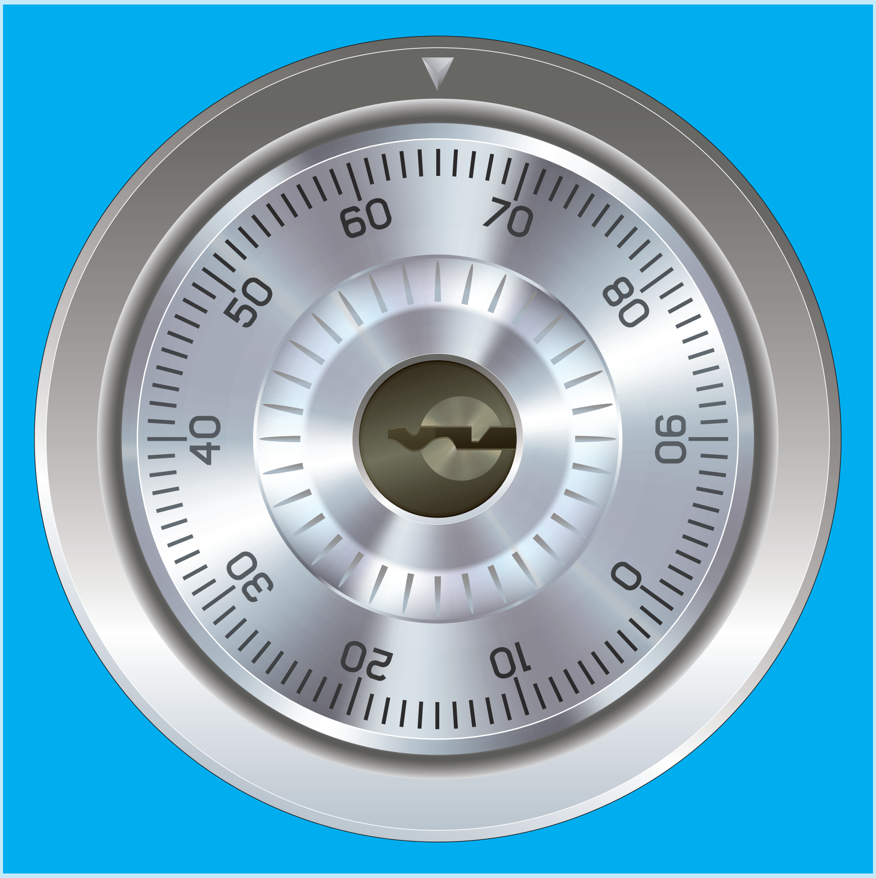
Feynman also discovered that many employees kept their locks on the factory settings or wrote the combination down! If employees kept their safes open while working, despite being instructed not to, Feynman could work out the last two numbers.

Another tactic Feynman used was to consider what numbers a physicist (or a mathematician) might use. One colleague used the first six digits of the mathematical constant e, the base for natural logarithms. Since $e = 2.718\ 281...$, his combination was 27 18 28.

BE PASSWORD WISE

I would not go past this opportunity to consider the use of passwords, which everyone uses in this electronic age. There are criminals out there with far less innocent motives than Feynman, who make their living by cracking passwords.

Students may consider some obvious guidelines, such as not writing passwords down and leaving them in obvious places, not using obvious passwords like your birthday, or even common words, and using a wide mixture of characters (letters,



A typical double digit combination lock.

numbers, symbols etc.) to increase the number of possibilities. Unlike Feynman, criminals can test millions of combinations per second (possibly much more), one reason why banks and other organisations close down a website after a certain number of tries. It is unbelievable that in a project of such high security, neither the scientists nor those in charge of the project ensured that every precaution was taken.

AN INTERESTING 'FRIEND'

There may have been other tactics he used, I will conclude with an interesting anecdote.

Klaus Fuchs was a friend who frequently lent Feynman his car to enable him to visit his critically ill wife in Albuquerque. However, when asked about possible spies, Fuchs mentioned Feynman's safe-cracking exploits and his frequent trips to Albuquerque (to see his wife). As a result, the FBI compiled a considerable dossier on Feynman. Interestingly, Fuchs would later confess to spying for the Soviet Union. Perhaps his real motive for lending the car was to draw attention away from himself.

FURTHER READING

https://en.wikipedia.org/wiki/Richard_Feynman

www.openculture.com/2013/04/learn_how_richard_feynman_cracked_the_safes_with_atomic_secrets_at_los_alamos.html

Feynman, R.P., (1985). *'Surely you're joking, Mr. Feynman': Adventures of a curious character.* W. W. Norton (US)

The relevant section is *Safecracker Meets Safecracker*, found in Part 3 *Feynman, the Bomb, and the Military.*

A very interesting further investigation is the cracking of the code for the German enigma machines during World War II, despite a calculation of 158,962,555,217,826,360,000 (nearly 159 quintillion) different settings. Like Feynman's combination locks, part of the success lay in the laxity of German officers, not to mention every message ending 'Heil Hitler'.

STIMULATING THINKING

Jessica Kurzman, Maths leader, St Patrick's Primary School

A picture sparks 1000 maths concepts! Use this picture as a prompt to stimulate thinking. If you have other ideas for investigations or lessons that could stem from the ideas here, add them to the conversation on our social channels. You can find us on Facebook and Instagram @maths.vic, LinkedIn @ maths-vic and on X, @maths_vic.

EARLY YEARS

- What are some things that you think would be bigger than this ship? What are some things that would be smaller? Can you think of anything that would be about the same size?
- Are there more red containers or more white containers? How do you know?
- Imagine you saw another ship that only had ten containers on it. Some containers were blue and some were red. How many might be blue and how many would be red? Can you draw those containers to show there are ten altogether?
- Imagine six people work on this ship. Can you draw six people? Can you draw six of something else? What would six look like on a dice? Can you find six of something and put them in a pile?
- What time or part of the day do you think this picture was taken? Explain why.
- Do you think this ship is light or heavy? Explain your thinking. Draw five things that might be lighter than this ship. Draw five things that might be heavier than this ship?
- What words could you use to describe the size of this ship?
- There are less than ten windows on this ship! How many windows might there be? Can you draw them and write the number? How many different possibilities can you come up with?
- Do you think this ship is moving fast or slow? What do you think might move slower than this ship? What do you think might move faster than this ship?
- If this ship was carrying cars, how many cars do you think would be in each container? Draw the cars you think would be in one container. How many cars is that? What would be one more? What would be one less?

FOUNDATION - YEAR 2

- This ship takes one whole day to travel home. What day might the ship have left, and what day would it arrive home?
- Describe the shape of the containers, without saying the name of the shape! Why do you think these containers are made in this shape and not other shapes?
- On this ship, there are 20 workers who come from Australia and New Zealand. How many workers do you think are from Australia and how many from New Zealand? How many different possibilities can you come up with? Choose one of your possible answers and represent it on a graph.
- Imagine there are 87 containers on the ship. How could you represent that number using materials so someone could easily count them?
- Imagine you fly a drone over the top of this ship and take a photo. Draw a picture of the photo.
- How many containers do you think are on this ship? Explain your answer.
- Imagine the containers on the ship need to be arranged in a repeating pattern. What might this pattern look like? Draw it and explain your pattern.
- Choose three containers on the back of the ship that are not in the same row. Put an 'x' on the square end of each of your choices. If the square end on each of the containers is one grid space, without using words, could you record the directions to move from one container to the next in the least amount of moves possible. Can you record the directions without using words?

YEARS 3 - 6

- Three ships leave Melbourne at different times on the same day. They all take four hours and 37 minutes to arrive at their destination. What time might each ship have departed and arrived? How many different possibilities can you come up with?
- The combined weight of four containers is two tonnes. What might the weight of each individual container be? How many different combinations can you come up with?
- In order to maintain the safe weight of the ship, each crew member is weighed upon boarding the ship. There were ten crew on this ship who each weighed between 65 and 68kg, but none of them were the same weight. What might each of their weights have been? Record them in order from lightest to heaviest.
- Ships measure their speed in knots. One knot is approximately 1.85km. We know this ship is travelling at between 20 and 25 knots per hour. How fast do you think this ship is travelling in knots per hour, and what would the equivalent in km per hour be? How many different possibilities can you come up with?
- There are eight crew members on this ship and their average age is 30 years old. How old might each of the crew members be? Is there more than one possible answer? Prove it.
- In each of the containers there are the same number of cars. How many cars do you think would fit in each container? If there are 157 containers in total on the ship, so how many cars would be on the ship altogether? How many different possibilities can you come up with?
- Apply the enlargement transformation to draw this picture so it is two times the current size.

YEAR 7 AND BEYOND

- Estimate the measurements (length, width and height) of one of the containers. Based on these measurements, calculate the surface area and the volume of the container. Then double the measurements and re-calculate the surface area and volume. What do you notice?
- The ship reported its departure time from Sydney, Australia as 1430hrs. What time would this be in five other cities in the world?
- Not all the containers have cargo in them. The ratio of empty to full containers is 1:4. If there are between 700 and 800 containers onboard, how many empty and full containers could there be on the ship? How many different solutions can you find?
- The containers are rectangular prisms. What other three-dimensional objects can you name and describe? For each of these, can you draw its net? Which of these would be suitable for carrying cargo on a ship, and which would not? Explain your reasoning, referring to the object's properties.
- It takes 25 workers 10 hours to unload the cargo off the ship, how many workers would be needed if the cargo needed to be unloaded in 1 hour?
- When arriving at customs, the authorities check a sample of the containers for prohibited items. Why would they not check every container and how could collecting information from a small random sample provide them with enough information to clear customs?
- Each time the crew disembark at a different port, they need to convert their money to a new currency. Find the current exchange rates for three currencies, and calculate the amount of money a crew member would receive if they converted 150 Australian dollars to that currency.

MAV education consultants can come to you and create a professional learning plan to build the capacity of teachers at your school. Reach out to our friendly team: primary@mav.vic.edu.au or secondary@mav.vic.edu.au.

MATRICES IN A SPREADSHEET

Andrew Stewart

In the Term 1 and 2 2024 editions of *Common Denominator*, Andrew explored matrices in a spreadsheet targeted towards the VCE General Mathematics. If you missed the first two parts, head to www.mav.vic.edu.au and download *Common Denominator*.

SHEET 8 (LESLIE)

A particular type of transition matrix, the Leslie matrix, can model the way population changes over time. The two key factors are the birth rate and survival rate for each age group. There are four basic models in which, over time, the population increases, decreases, plateaus or repeatedly cycles.

Information about a population of rodents is given in Table 1.

Age group (years)	0 - 1	1 - 2	2 - 3
Initial population	100	100	100
Birth rates	0.4	1.2	0.3
Survival rates	0.6	0.3	0

Table 1.

Set up a spreadsheet as shown in Table 2.

To calculate S1, select cells G2:G4. Enter: =MMULT(A2:C4,E2:E4). Press and hold the Control and Shift keys, and then press Return. To calculate S2, select cells H2:H4. Enter: =MMULT($A$2:$C$4,G2:G4).

Press and hold the Control and Shift keys, and then press Return. To calculate S3 to S5, select cells H2:H4 and FillRight to cells K2:K4. To calculate the total population for S1, select cell G5. Enter =SUM(G2:G4) and press Enter. To calculate the total populations for S2 to S5, select cell G5 and FillRight to cell K5.

The spreadsheet should now look like Table 3 (some values have been truncated/rounded due to space).

This matrix models a situation where the population is increasing.

Information about another population of rodents is given in the Table 4.

Enter the key values from the table into the spreadsheet and confirm the values shown in Table 5 (some values have been truncated/rounded due to space).

	A	B	C	D	E	F	G	H	I	J	K
1	L				S0		S1	S2	S3	S4	S5
2	0.4	1.2	0.3		100						
3	0.6	0	0		100						
4	0	0.3	0		100						
5						sum					

Table 2.

	A	B	C	D	E	F	G	H	I	J	K
1	L				S0		S1	S2	S3	S4	S5
2	0.4	1.2	0.3		100		190	157	205	205	238
3	0.6	0	0		100		60	114	94.2	123	123
4	0	0.3	0		100		30	18	34.2	28.3	36.9
5						sum	280	289	333	357	398

Table 3.

Age group (years)	0 - 1	1 - 2	2 - 3
Initial population	100	100	100
Birth rates	0.3	1.2	0.25
Survival rates	0.5	0.3	0

Table 4.

	A	B	C	D	E	F	G	H	I	J	K
1	L				S0		S1	S2	S3	S4	S5
2	0.3	1.2	0.25		100		175	120	145	122	128
3	0.5	0	0		100		50	87.5	60	72.4	61
4	0	0.3	0		100		30	15	26.3	18	21.7
5						sum	255	223	231	212	211

Table 5.

This matrix models a situation where the population is decreasing.

To model a repeated cycle, the Leslie matrix must generate an Identity matrix when raised to the power of its order. For a 3 x 3 Leslie matrix the power will be 3.

Set up the following lower down the same spreadsheet page as shown in Table 6.

To calculate L^2, select cells I11:K13. Enter: =MMULT(A11:C13,A11:C13).

Press and hold the Control and Shift keys, and then press Return. To calculate L^3 (L x L^2), select cells E11:G13. Enter: =MMULT(A11:C13,I11:K13).

Press and hold the Control and Shift keys, and then press Return. The spreadsheet should now look like Table 7.

Matrix calculations are not as easy in the spreadsheet as they are in the calculator, hence the two-step process for dealing with powers.

Enter the values from cells A11:C13 into cells A2:C4. Select cells K2:K5 and FillRight to cells L2:L5.

Continue the state labels to S6 in cell L1.

Adjust the S0 values to those shown in Table 8 and confirm the state matrix values.

This matrix (as expected) models a situation where all the population values are cycling or oscillating on a three-year cycle.

Change the values in cells B2 and C2 to those shown in Table 9.

Select cells L2:L5 and FillRight to cells Z2:Z5.

Continue the state labels from S7 in cell M1 to S20 in cell Z1.

Adjust the S0 values to those shown in Table 9 and confirm the state matrix values (some values have been truncated/rounded due to space).

This matrix models a situation where all the population values are plateauing off.

For larger order Leslie matrices, the spreadsheet template for finding the Identity matrix becomes a little more complicated

The template in Table 10 works for a 4 x 4 Leslie matrix to generate a cyclic situation.

To calculate L^2, select cells K2:N5. Enter: =MMULT(A2:D5,A2:D5).

Press and hold the Control and Shift keys, and then press Return. To calculate L^4 (L^2 x L^2), select cells F2:I5. Enter: =MMULT(K2:N5,K2:N5). Press and hold the Control and Shift keys, and then press Return.

The cycle created repeats itself every four states.

The L matrix could be used as a starting point to find a matrix that generates a plateauing sequence.

	A	B	C	D	E	F	G	H	I	J	K
1	L				L^3				L^2		
2	0	0	4								
3	0.5	0	0								
4	0	0.5	0								
5											

Table 6.

	A	B	C	D	E	F	G	H	I	J	K
1	L				L^3				L^2		
2	0	0	4		1	0	0		0	2	0
3	0.5	0	0		0	1	0		0	0	2
4	0	0.5	0		0	0	1		0.25	0	0
5											

Table 7.

	A	B	C	D	E	F	G	H	I	J	K	L
1	L				S0		S1	S2	S3	S4	S5	S6
2	0	0	4		20		40	20	20	40	20	20
3	0.5	0	0		10		10	20	10	10	20	10
4	0	0.5	0		10		5	5	10	5	5	10
5						sum	55	45	40	55	45	40

Table 8.

	A	B	C	D	E	F	G	H	I	...	Y	Z
1	L				S0		S1	S2	S3		S19	S20
2	0	1	2		20		30	20	25		24.0	24.0
3	0.5	0	0		10		10	15	10		12.0	12.0
4	0	0.5	0		10		5	5	7.5		6.0	6.0
5						sum	45	40	42.5		42.0	42.0

Table 9.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	L					L^4					L^2			
2	0	0	0	4		1	0	0	0		0	0	0.8	0
3	0.5	0	0	0		0	1	0	0		0	0	0	2
4	0	2.5	0	0		0	0	1	0		1.25	0	0	0
5	0	0	0.2	0		0	0	0	1		0	0.5	0	0

Table 10.

ONE MINUTE WITH TOAN HUYNH

I'M...

Toan (pronounced Twan!) and I'm the Director of Teaching and Learning at CS in Schools. We're a charity aiming to grow the number of skilled technology graduates to fill the ever increasing demand for computer scientists and software engineers in the industry. I design, produce and roadtest student courses and teacher training programs we offer for free to all schools Australia wide!

I CHOSE A CAREER IN MATHEMATICS EDUCATION...

Because I love maths! And I love how computer science brings maths to life by taking handwritten equations from exercise books and translating them to creating characters in virtual worlds, or applying it to understand and solve real world problems. Sharing and fostering this enthusiasm to young people so they can grow into the next wave of smart and productive Australian's is such a privilege.

AT UNI I STUDIED…

Computer science and computer systems engineering. A part of me wishes I studied pure mathematics too!

MY CAREER HAS TAKEN ME ON QUITE A JOURNEY…

I started off as a coder at ANZ bank, then became a video games developer making racing and platformer games for Xbox and Playstation. I worked in the US for Microsoft, Xbox and eBay which was a fantastic professional and personal experience. I finally saw the light (!) and moved across to teaching both tertiary and secondary students back home in Australia.

CS IN SCHOOLS MAKES A DIFFERENCE …

By making a difference to teachers. Computer science and coding is a relatively new field that it can be associated with a lack of confidence and an understandable reluctance to learn and teach the subject. Our courses and training aim to improve on this by gently scaffolding content, providing industry support and focusing on the process of learning first and product second. We strengthen the connections between maths and computer science. I believe if you're a maths teacher, you'd make a great computer science teacher. Computer science started life as applied mathematics.

AT EBAY I LEARNED A LOT…

In particular, how important it is to to commercial organisation to keep track of and understand data. We made millions of dollars by reviewing graphs and charts each month and tweaking decisions and features on the website based on what the information was telling us. Another example of maths and technology working together in the real world.

STUDENTS NEED TO BUILD DIGITAL CAPABILITIES…

To effectively and efficiently solve problems in the future, understanding, using and producing new digital tools is imperative. It's not just about using digital technologies to keep up, it's about understanding the foundations of what drives digital innovations and technologies (hint: maths has a big role here) so we can lead the production of new Australian-made technologies. I'd love to shift our state from being reactive consumers of digital products to proactive innovators of new digital technologies.

TEACHERS CAN….

Make a big difference to their students by modelling how they are also learning and growing, and sharing their successes and lessons along the way! We model how to solve simultaneous equations on the board very well. Let's model how we solve some of our own real-life problems with maths too.

I THRIVE WHEN….

I see an interesting phenomenon in the world and can see a clear pathway to breaking it down and making it understandable to others.

MAC OR PC?....

PC. I find them easier to code on.

MY LITTLE SECRET...

I could eat banh mi for lunch every day without fail.

BOOK REVIEW: OPEN MIDDLE MATH

Larissa Raymond – Mathematics education consultant, MAV

In 2023, MAV was fortunate to host a number of face-to-face professional learning opportunities in collaboration with Robert Kaplinsky. They were a great success and the reason has a lot to do with Kaplinsky's pedagogical approach; the ways in which he co-creates safe, active learning spaces where all are likely to experience a sense of belonging and connection.

The second reason for the success of the workshops is predicated on Kaplinsky's mathematical content knowledge and using this knowledge to design rich and rigorous mathematical learning experiences that foster the development of deep critical thinking, analytical reasoning and problem solving. In other words, designing tasks that actively foster the four mathematical proficiencies: understanding, reasoning, problem solving and fluency and, productive dispositions.

The importance of developing these proficiencies and productive dispositions have been further amplified in the recently updated Victorian Mathematics Curriculum (v2) and can also be found in the Australian Curriculum.

Kaplinsky's book, *Open Middle Math: Problems that unlock student thinking, Grades 6-12*, explores how to co-design, with colleagues, rich open middle problems that foster evidence informed, contemporary pedagogical approaches, productive dispositions and the co-creation of learning environments where all; young people and teachers, are likely to experience connection, curiosity and a sense of joy.

Kaplinsky creatively interconnects the importance between a teacher's practice and their content knowledge and the implications this has on young people's mathematical learning, sense of self and agency, as he explores the power of open middle problems.

Kaplinsky describes open middle problems as mathematical problems that are,

> '...characterised by a closed beginning, - meaning all students start with the same initial problem, and a closed end - meaning there is only one correct or optimal answer. The key is that the middle is open - in the sense that there are multiple ways to approach and ultimately solve the problem.'

Throughout the book, Kaplinsky provides a number of open middle problems to explore with students in Years 6 through to Year 12. Each problem is carefully scaffolded to support the reader in understanding how to take a typical 'textbook' closed problem and turn it into a number of open middle problems, requiring varying degrees of critical thinking, analytical reasoning and problem solving. After doing this, a detailed reflection outlining what students may or may not do and how you might respond, is provided.

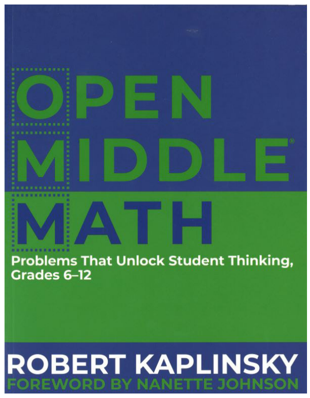
As well as providing carefully curated open middle problems, Kaplinsky draws on the seminal work of authors Smith and Stein (2011), to support teaching teams in not only furthering their understanding of the critical elements of great planning (or designing) for optimal learning, but also the necessary knowledge and know how to translate this into their practice in recognition of each school's unique context.

As Kaplinsky scaffolds this design process, he shares a number of dynamic pedagogical manoeuvres and how these can be slowly and intentionally translated into every teacher's everyday practices and ways of being.

One of these pedagogical manoeuvres includes facilitating powerful learning conversations. Kaplinsky explores the value of using conversation prompts and questions to support rigorous 'mathematical learning conversations' during small group and whole class discussions involving open middle problems; that is, conversations that promote the benefits of learning with and from others.

He also provides a window into the types of questions that are likely to support the activation of the four proficiencies and productive dispositions.

Kaplinsky then goes on to highlight how carefully crafted questions and young people's responses can not only reveal misconceptions, but also how best to use these insights to promote further questioning that is more likely to enable the learner to understand their misconceptions and how they might approach the task differently.



Open Middle Math is available to purchase via the MAVshop. Visit www.mav.vic.edu.au/mav-shop.

Through his own experimentation with questioning, Kaplinsky says he was able to see young people's misconceptions '...more clearly and use them as talking points to strengthen mathematical understandings...'.

Kaplinsky also brings attention to how teachers can further strengthen their capacity to:

- uncover and explore students' thinking.
- evaluate how young people build conceptual understanding.
- foster flexibility, creativity and, the power of reflection.
- co-create the conditions to enable all learners to be and become discerning thinkers.
- co-create dynamic learning spaces that minimise anxiety and fear.
- use and design open middle problems that have a low floor and high ceiling.

Open Middle Math: Problems That Unlock Student Thinking, Grades 6-12 is a wonderful teaching resource that will certainly support teaching teams as they continue to create the conditions and opportunities for all learners to thrive in being and becoming great mathematics learners.

MATHEMATICAL LANGUAGE

Di Liddell - Education manager, MAV

THE IMPORTANCE OF MATHEMATICAL LANGUAGE DEVELOPMENT IN EARLY CHILDHOOD EDUCATION

Acquiring mathematical language significantly shapes a child's adeptness in comprehending and expressing mathematical concepts. Early childhood education lays the groundwork for this by nurturing mathematical vocabulary, a pivotal foundation for a strong grasp of mathematical principles. This article delves into the vital aspects of developing mathematical language in young learners. It explores the utilisation of number talks, interactive word walls, and picture storybooks as strategies that effectively foster mathematical language proficiency and enhance conceptual understanding in children.

The acquisition of mathematical language is intricately linked to cognitive development in young children. Drawing from Piaget's cognitive developmental theory, children construct knowledge through interactions with their environment. Language serves as a conduit for children to assimilate mathematical concepts. As children acquire mathematical terms, they grasp abstract ideas and build cognitive frameworks that underpin more sophisticated mathematical thinking (Gelman & Gallistel, 1978).

The cultivation of a robust mathematical vocabulary is pivotal in enabling children to articulate mathematical concepts accurately. Through exposure to mathematical language in contextually relevant scenarios, children learn terms such as addition, subtraction, equal, greater than, less than, etc. This vocabulary enrichment empowers them to describe, explain, and reason mathematically, bolstering both comprehension and communication skills (Baroody & Wilkins, 1999)

A fundamental aspect of successful mathematical problem-solving lies in effective communication. Proficiency in mathematical language enables children to express their reasoning, share strategies, and collaborate with peers (Clarke, 2003). This proficiency cultivates an atmosphere where diverse problem-solving approaches can be discussed and explored, thereby nurturing critical thinking skills.

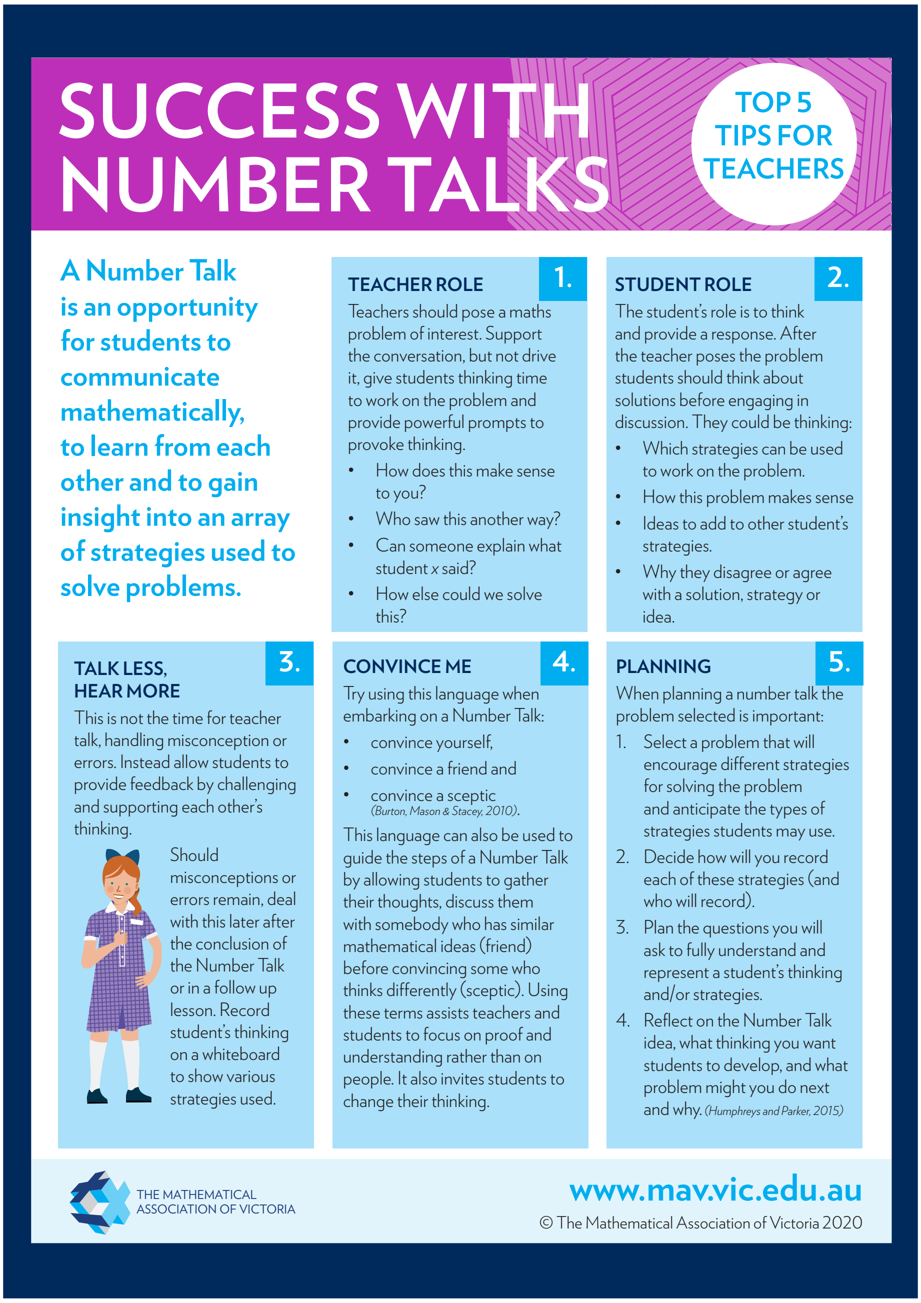


Figure 1. Teacher tip sheet sharing tips on number talks. MAV members can download a printable version of this for the classroom. Visit http://bit.ly/number_talks_success

Both educators and parents play crucial roles in developing these mathematical language skills (Ginsburg et.al., 2010). They can create enriching learning settings that promote mathematical discussions, provide opportunities for activities rich in vocabulary, and serve as models for using mathematical language.

Now, let's explore how strategies such as number talks, interactive word walls, and picture storybooks serve as effective approaches in developing mathematics language.

NUMBER TALKS

Number talks are structured discussions or routines in mathematics classrooms that focus on mental maths strategies and encourage students to share their problem-solving approaches for numerical tasks.

Number talks aim to enhance students' mathematical language development by fostering their ability to articulate their reasoning, understand diverse perspectives, and communicate mathematical ideas effectively.

Through these discussions, students engage in mathematical discourse, refine their mathematical thinking, and develop a deeper understanding of number relationships and operations.

Number talks promote a supportive environment where students learn from each other's strategies, leading to increased confidence and proficiency in mathematical concepts. Empirical studies, such as those by Parrish (2010) and Hufferd-Ackles et al. (2017), have highlighted the positive impact of number talks on students' mathematical understanding and communication skills. See Figure 1.

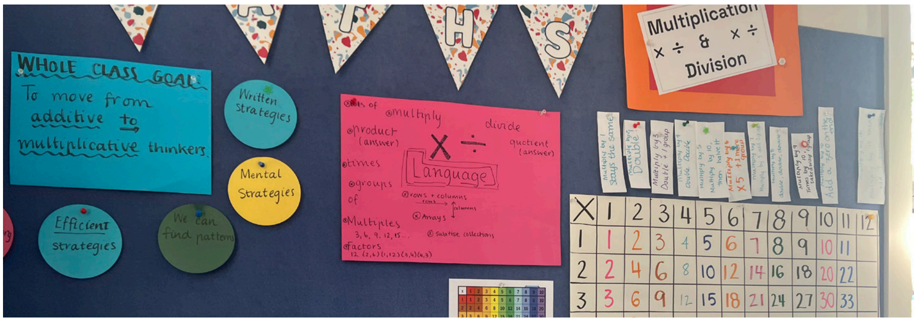
INTERACTIVE WORD WALLS

Interactive mathematics word walls are dynamic classroom displays containing mathematical vocabulary, concepts, symbols, and visual representations that serve as a reference and learning tool for students. Word walls aim to support mathematic language development by providing a visual and interactive environment where students can explore, internalise, and apply mathematical language in context.

Through regular interaction with the displayed words and concepts, students develop a deeper understanding of mathematical terms, connections between different mathematical ideas, and the ability to communicate their mathematical thinking effectively. By engaging with the word wall collaboratively and using it as a resource during discussions and problem-solving activities, students enhance their mathematical language skills and conceptual understanding. Research by Tovani and Crawford (2012) and Chapman (2017) underscores the benefits of interactive mathematics word walls in fostering mathematical language development and improving student comprehension and communication in mathematics.

PICTURE STORYBOOKS

Picture story books play a pivotal role in enhancing mathematical language development by integrating mathematical concepts within engaging narratives and visual representations.



These books offer opportunities for students to explore mathematical ideas in context, fostering a deeper understanding of mathematical concepts while simultaneously developing their language skills. Through stories that incorporate mathematical themes, characters, and situations, students encounter mathematical vocabulary and concepts in meaningful contexts, allowing for a more profound comprehension and application of mathematical language. Visual illustrations provide concrete representations of abstract mathematical ideas, aiding in the visualisation and understanding of mathematical concepts. Studies by Lamon (2007) and Ginsburg (2009) emphasise the significance of picture story books in promoting mathematical language development, conceptual understanding, and fostering positive attitudes towards mathematics.

CONCLUSION

The development of mathematical language in early childhood education serves as a cornerstone for a child's adeptness in comprehending and articulating mathematical concepts.

Through the acquisition of a diverse mathematical vocabulary, children not only grasp mathematical principles but also effectively communicate their thoughts and rationale. Educators and parents play pivotal roles in fostering an environment conducive to mathematical language growth, ultimately supporting children in becoming proficient mathematical thinkers.

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MAV has many mathematically rich picture story books available in the MAVshop, visit www.mav.vic.edu.au/Resources/Primary-resources/Picture-books to download a resource mapping individual titles to the Australian Curriculum mathematics strands.

LEVERAGING THE MATILDA'S

Jack Fray - STEM specialist, Bell Primary School

Anecdotal evidence tells us that since the 2023 World Cup was held in Australia, the number of women and girls participating in soccer at a club level has doubled. The national pride felt towards athletes representing the country hit a record high (an almost 200% increase in just a single month). And kids of all genders would simply not stop talking about it.

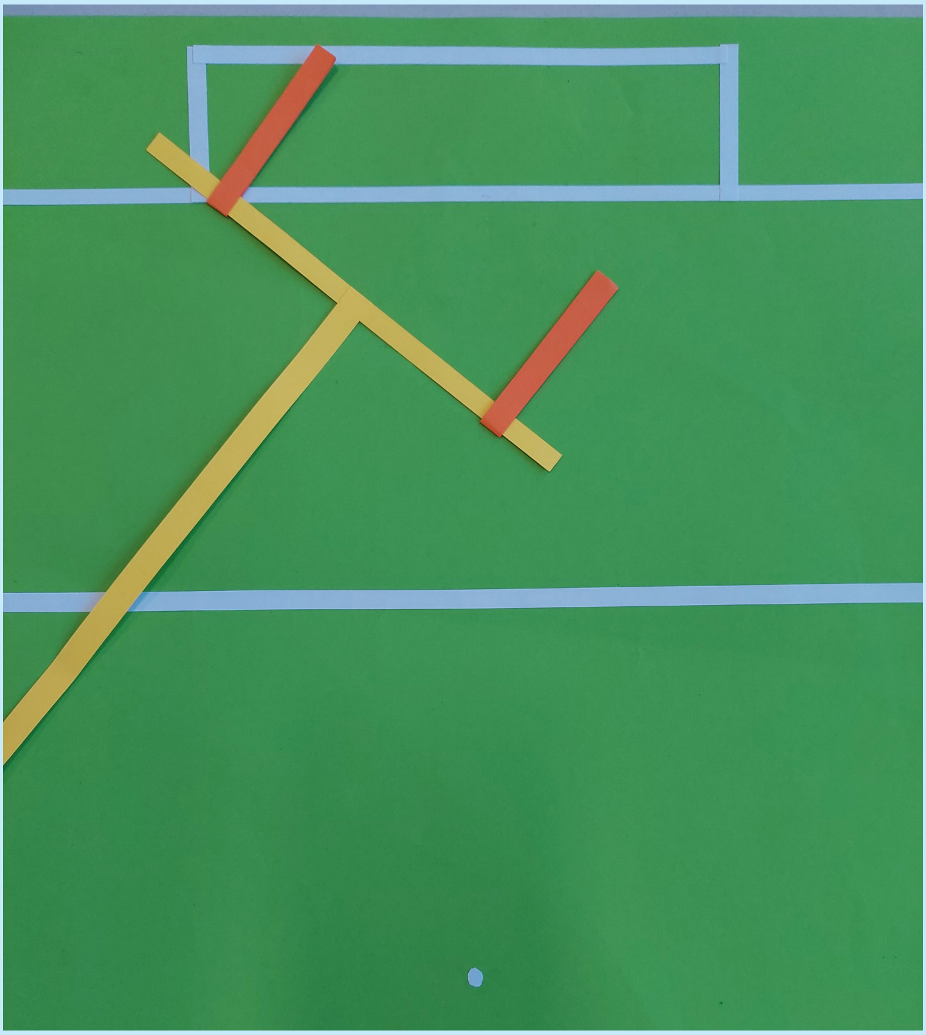
This was an opportunity to connect mathematics with the real world that, like a Cortnee Vine penalty, was too good to miss. Working with a Year 6 cohort, we created the shot-o-meter, a device used to measure how much of the goal was visible to a striker when attacking on an angle.

But before we spoke about complimentary and reciprocal angles, or adjusted the callipers to demonstrate the relative size of the goal, we headed out to the oval with an oversized protractor and measuring tapes to get some real-life experience of shooting the ball and recording our successful kicks, much like Doug Clarke's famous 'footy angles' lesson. This gave us the chance to carry out some statistical analysis and verify our hunch that the tighter the angle, the more difficult the shot.

Back in the classroom, once the data had been sifted, students turned their attention to investigating why it's harder to score from a tighter angle, rather than dead in front, with the shot-o-meter providing a concrete model of how the target gets visibly narrower as the striker moves to the side.

Once the assertion had been proven that the penalty spot was indeed the best place to shoot from, the Australia vs France game gave the next point of interest, sparking a conversation (and an investigation) about the percentage of successful penalty kicks taken over the course of the tournament. Which in turn led us to question some commentator's wisdom, namely that the most likely time to concede a goal was within five minutes of scoring one, and that the most likely time to score was just before half time and full time. With all the statistical information we needed readily available (thanks Wikipedia!) students begun to design their own methods for analysis.

The Olympics have further extended our interest in sport and mathematics. My class has more sporty investigations underway!



Australia	0–0 (a.e.t.)	France
	Report	
	Penalties	
Foord ✔	7–6	✘ Bacha
Catley ✘		✔ Diani
Kerr ✔		✔ Renard
Fowler ✔		✔ Le Sommer
Arnold ✘		✘ Périsset
Gorry ✔		✔ Geyoro
Yallop ✔		✔ Karchaoui
Carpenter ✔		✔ Lakrar
Hunt ✘		✘ Dali
Vine ✔		✘ Bècho

SMART GOALS IN MATHEMATICS

Jennifer Sze, The University of Melbourne

In the insightful podcast, *Numeracy Guide Level 3 to 8 Examples of how to use the HITS with a focus on developing students' numeracy*, Jennifer Bowden underscores the vital role of engaging students in meaningful discussions to ensure they not only comprehend but also can realistically achieve their set goals. Through these dialogues, teachers wield language strategically, fostering active engagement with learning objectives. This active participation extends to peer and self-assessment, enabling students to effectively monitor their progress and development.

Jen emphasised the collaborative creating of visual aids, displays, or 'I can' statements, as tangible benchmarks for individual success within the classroom. Such visual representations empower students to identify achieved goals and strategically plan next steps in the learning journey. Jen discussed the necessity of understanding students' behaviours, actions, and learning processes, advocating for a holistic assessment approach that encompasses all domains of learning. Bowden elucidates how these goal-setting practices seamlessly integrate into classroom dynamics and are adaptable across various subjects and environments, whether in specialist areas or within students' homes. This adaptability underscores the universality and practicality of the goal-setting framework (Buzza & Dol, 2015).

ELEMENTS OF SETTING GOALS

- Based on assessed student needs
- Goals are presented clearly so students know what they are intended to learn
- Focus on surface and/or deep learning
- Challenges students relative to their current mastery of the topic
- Links to explicit assessment criteria.

In a study by Bostwick et al. (2017), they discovered that growth mindset, self-based growth goals, and task-based growth goals were well represented by an underlying growth orientation factor. To ensure students achieve success in their learning in mathematics, teachers might want to consider the following reflective prompts:

- What are the key steps in setting effective numeracy goals?
- How can you ensure that the numeracy learning goals you set are clearly communicated to the students?
- How can you set realistic goals to challenge all students in your classroom?
- How do you encourage students to engage actively in planning goals?
- Explain why it is important to link numeracy goals to assessment.

PUTTING IT INTO ACTION

Referring to setting goals for students to achieve success in mathematics, I reflect on my time as a middle years literacy and numeracy support teacher, where I was responsible for Year 8-10 students, who were at risk of not completing their study due to academic performance well below the standard level. For context, these students came from a diverse cultural and religious backgrounds. My goals with these students was to form a trusting relationship and practise culturally responsible pedagogy (Howard, 2022). I was sensitive to the culture of my students and where possible, weaved stories from their respective countries into my teaching.

I guided the students in setting their precise daily learning objectives, with the overarching Specific, Measurable, Achievable, Relevant and Timely (SMART) goals of enriching their involvement and confidence in mathematics. By involving students in crafting their learning objectives, I empowered the class with a sense of ownership and agency over their educational journey (Sides & Cuevas, 2020). As the objectives evolved in clarity and specificity, I noticed the shift in students' attitudes and behaviours towards mathematics learning. The goal-setting approach did bolster students' commitment to achievement-oriented behaviour.

THE EFFECTS OF GOAL SETTING

Goal setting underpins the vital role of growth mindset as advocated by Professor Jo Boaler. Boaler's growth mindset in mathematics (2019) is based on the idea that intelligence and mathematical ability are not fixed traits.

These traits can be developed through careful intervention by teachers through effort and supportive learning environment. Literature has shown that growth mindset and setting goals in mathematics empowers students to develop their abilities and to achieve their fullest potential.

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Listen to the podcast episode at: https://bit.ly/SMART-Goals-success

MAV's mathematics education consultants can work with your school to set SMART goals for your unique context. Reach out to our friendly team: primary@mav.vic.edu.au or secondary@mav.vic.edu.au.

COLLABORATIVE PL

Claire Embregts, Community strategy manager, MAV

IMPORTANCE OF COLLABORATIVE PROFESSIONAL LEARNING

Collaborative professional learning (CPL) stands out from traditional professional development in the ever-changing education landscape. Unlike the solitary learning experiences often associated with traditional professional development, CPL fosters a profound sense of community among educators. This approach encourages teachers to engage in shared learning experiences, collectively exploring new ideas, discussing challenges, and developing innovative solutions. The collaborative nature of CPL not only enriches individual professional growth but also builds a strong, supportive network that promotes continuous improvement, making educators feel connected and supported.

The benefits of CPL are wide ranging. It leverages the collective expertise of educators, allowing them to learn from each other's experiences and insights. A shared knowledge pool can lead to the development of more effective teaching strategies, which can be directly applied in the classroom. CPL significantly breaks down teachers' isolation, especially in challenging teaching environments.

By participating in a collaborative learning community, teachers not only gain access to a wealth of resources but also a sense of belonging and mutual support. This can significantly reduce burnout and relieve stress, making educators feel less overwhelmed and more resilient.

OVERVIEW OF THE IMPACT OF COMMUNITY ON TEACHING PRACTICE

A strong community of educators can profoundly impact teaching practice. When teachers are part of a vibrant professional community, they have access to a wealth of resources and support that can enhance their instructional methods and classroom management skills. Community-driven initiatives, such as resource sharing, peer feedback, and collaborative planning, provide practical benefits that empower educators and improve teaching quality.

Resource sharing within a community allows educators to access diverse teaching materials, lesson plans, and innovative tools they might have yet to discover independently. This can lead to more engaging and effective classroom experiences for students. Receiving constructive feedback from peers helps teachers refine their practices and address any areas of improvement more effectively.

Real-life examples underscore the positive impact of community engagement. Teachers actively participating in their professional communities often report higher job satisfaction and professional fulfilment levels. They also adopt more reflective and adaptive teaching practices, improving student outcomes. In essence, a robust community supports teachers in their professional journey and fosters an environment where continuous learning and improvement are the norms.

The following sections will explore specific examples of how these strategies can be successfully implemented and discuss future directions and innovations for our community.

LEVERAGING TECHNOLOGY FOR COLLABORATION

MAV's closed online community platform plays a crucial role in facilitating collaborative professional learning. It provides a secure and dedicated space for educators to connect, share resources, and engage in ongoing discussions, making them feel more connected and engaged in the learning process.

To maximise the platform's features, educators should actively participate in discussion forums to ask questions, share insights, and engage in meaningful conversations with fellow educators. Utilising the resource libraries can provide access to various teaching materials, lesson plans, and professional development resources shared by other members.

Contributing to community content enhances professional visibility. Writing and sharing blog posts or articles on topics of interest or expertise contributes to the community and enhances professional visibility. Sharing case studies or success stories highlights how collaborative learning strategies have been applied in teaching practice, inspiring and informing other members.

PROMOTING A CULTURE OF SHARING AND FEEDBACK

Creating a culture where sharing and feedback are valued is essential for the success of collaborative professional learning. Educators openly sharing their experiences, resources, and ideas can lead to richer learning experiences and more effective teaching practices.

Recognising and celebrating the contributions of community members who actively share and provide feedback fosters a supportive environment. Encouraging and valuing constructive feedback further enriches the learning experience.

By integrating these elements within our closed online community, we can create a dynamic and supportive environment that fosters collaborative professional learning. It also enhances individual educators' professional growth and strengthens our community's overall teaching practice.

Join us in this endeavour! Together, we can shape a community that not only grows in numbers but thrives in the richness of diverse interactions.

If you're a maths educator looking to grow your skills, expand your network, and access valuable resources, you can join MAV's community at www.mav.vic.edu.au/Membership/Community.

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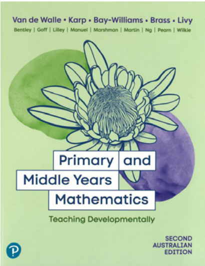
MAV MEMBERS GET A DISCOUNT ON ALL STOCK

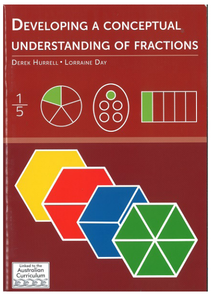


PRIMARY AND MIDDLE YEAR MATHEMATICS (SECOND EDITION)

F-9

An Australian-adapted text that is designed to support teachers in implementing research-informed approaches for teaching mathematics. This comprehensive resource aims to enhance teachers' knowledge of effective teaching approaches, encompassing assessment for learning, understanding students' learning processes in mathematics, and addressing common misconceptions. Each chapter refers to the Australian Mathematics Curriculum and presents classroom-ready, hands-on problem-solving tasks specifically designed to challenge students' thinking and foster positive attitudes towards mathematics. New to this edition: The Mathematics Learning Area of AC 9.0 is significantly different to AC 8.4. It has a simplified structure, and descriptions for year levels, achievement standards, content descriptions and content elaborations have all changed – all with the aim of enriching student learning experiences. This edition explains the 'what' and the 'why' and shows how the content aligns to AC 9.0.

$140 (MEMBER)
$175 (NON MEMBER)



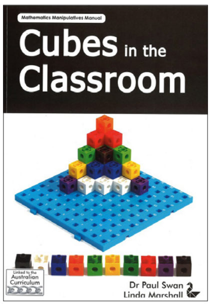
DEVELOPING A CONCEPTUAL UNDERSTANDING OF FRACTIONS

F-6

This publication is built on the idea that there is a difference between a conceptual understanding of fractions and the capacity to operate (do sums!) with fractions. An ability to operate with fractions doesn't always show or lead to an understanding of fractions, but an understanding of fractions almost invariably contributes towards acquiring proficiency in the operations of fractions.

Many teachers and researchers will assert that as a topic, fractions is one which is challenging, but necessary, to teach and learn. It is a topic with complex connections to other areas of mathematics and seems to carry many misconceptions. The book focuses on the teaching and learning of common fractions and connections are made to decimal fractions, percentages and ratios.

$29.25 (MEMBER)
$36.55 (NON MEMBER)



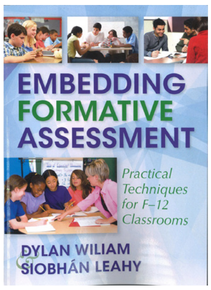
CUBES IN THE CLASSROOM

1-6

Linking or connecting cubes may be used for early years construction, particular with the addition of extra piece such double cubes, joiners and half cubes.

2cm linking cubes may then be used later in the school to introduce number ideas, algebraic thinking, measurement ideas such as volume and spatial ideas involving viewing cube constructions from different angles. This book by Dr Paul Swan will help teachers gain the most from the existing 1cm and 2cm cubes found in most storerooms and get the cubes into classrooms.

$22.10 (MEMBER)
$27.60 (NON MEMBER)



EMBEDDING FORMATIVE ASSESSMENT

F–VCE

Effective classroom formative assessment helps educators make minute-by-minute, day-by-day instructional decisions, but putting it into practice requires both a robust collection of techniques and an understanding of how to use them. This book is a clear, practical guide for teachers, centred on five key strategies for improving teacher practice and student achievement.

An overview of each strategy is provided and a number of very practical formative assessment techniques for implementing it in F–12 classrooms. Along with guidance on when and how to use the specific techniques, they provide tips, cautions and enhancements to sustain formative assessment. A student reflection form, peer observation form and self-reflection checklist accompany each strategy.

$40.50 (MEMBER)
$50.60 (NON MEMBER)

THE MATHEMATICAL ASSOCIATION OF VICTORIA

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